Chapter 6

Inventory Control Models
Learning Objectives

After completing this chapter, students will be able to:

1. Understand the importance of inventory control and ABC analysis.
2. Use the economic order quantity (EOQ) to determine how much to order.
3. Compute the reorder point (ROP) in determining when to order more inventory.
4. Handle inventory problems that allow quantity discounts or non-instantaneous receipt.
Learning Objectives

After completing this chapter, students will be able to:

5. Understand the use of safety stock.
6. Describe the use of material requirements planning in solving dependent-demand inventory problems.
7. Discuss just-in-time inventory concepts to reduce inventory levels and costs.
8. Discuss enterprise resource planning systems.
Chapter Outline

6.1 Introduction
6.2 Importance of Inventory Control
6.3 Inventory Decisions
6.4 Economic Order Quantity: Determining How Much to Order
6.5 Reorder Point: Determining When to Order
6.6 EOQ Without the Instantaneous Receipt Assumption
6.7 Quantity Discount Models
6.8 Use of Safety Stock
Chapter Outline

6.9  Single-Period Inventory Models
6.10 ABC Analysis
6.11 Dependent Demand: The Case for Material Requirements Planning
6.12 Just-in-Time Inventory Control
6.13 Enterprise Resource Planning
Introduction

- Inventory is an expensive and important asset to many companies.
- Inventory is any stored resource used to satisfy a current or future need.
- Common examples are raw materials, work-in-process, and finished goods.
- Most companies try to balance high and low inventory levels with cost minimization as a goal.
  - Lower inventory levels can reduce costs.
  - Low inventory levels may result in stockouts and dissatisfied customers.
Introduction

- All organizations have some type of inventory control system.
- Inventory planning helps determine what goods and/or services need to be produced.
- Inventory planning helps determine whether the organization produces the goods or services or whether they are purchased from another organization.
- Inventory planning also involves demand forecasting.
Introduction

Inventory planning and control

Planning on What Inventory to Stock and How to Acquire It

Forecasting Parts/Product Demand

Controlling Inventory Levels

Feedback Measurements to Revise Plans and Forecasts

Figure 6.1
Importance of Inventory Control

- Five uses of inventory:
  - The decoupling function
  - Storing resources
  - Irregular supply and demand
  - Quantity discounts
  - Avoiding stockouts and shortages

- Decouple manufacturing processes.
  - Inventory is used as a buffer between stages in a manufacturing process.
  - This reduces delays and improves efficiency.
Importance of Inventory Control

- Storing resources.
  - Seasonal products may be stored to satisfy off-season demand.
  - Materials can be stored as raw materials, work-in-process, or finished goods.
  - Labor can be stored as a component of partially completed subassemblies.

- Compensate for irregular supply and demand.
  - Demand and supply may not be constant over time.
  - Inventory can be used to buffer the variability.
Importance of Inventory Control

- Take advantage of quantity discounts.
  - Lower prices may be available for larger orders.
  - Extra costs associated with holding more inventory must be balanced against lower purchase price.

- Avoid stockouts and shortages.
  - Stockouts may result in lost sales.
  - Dissatisfied customers may choose to buy from another supplier.
Inventory Decisions

- There are two fundamental decisions in controlling inventory:
  - How much to order.
  - When to order.

- The major objective is to minimize total inventory costs.

- Common inventory costs are:
  - Cost of the items (purchase or material cost).
  - Cost of ordering.
  - Cost of carrying, or holding, inventory.
  - Cost of stockouts.
# Inventory Cost Factors

<table>
<thead>
<tr>
<th>ORDERING COST FACTORS</th>
<th>CARRYING COST FACTORS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Developing and sending purchase orders</td>
<td>Cost of capital</td>
</tr>
<tr>
<td>Processing and inspecting incoming inventory</td>
<td>Taxes</td>
</tr>
<tr>
<td>Bill paying</td>
<td>Insurance</td>
</tr>
<tr>
<td>Inventory inquiries</td>
<td>Spoilage</td>
</tr>
<tr>
<td>Utilities, phone bills, and so on, for the purchasing department</td>
<td>Theft</td>
</tr>
<tr>
<td>Salaries and wages for the purchasing department employees</td>
<td>Obsolescence</td>
</tr>
<tr>
<td>Supplies, such as forms and paper, for the purchasing department</td>
<td>Salaries and wages for warehouse employees</td>
</tr>
<tr>
<td>Utilities and building costs for the warehouse</td>
<td></td>
</tr>
<tr>
<td>Supplies, such as forms and paper, for the warehouse</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.1
Inventory Cost Factors

- Ordering costs are generally independent of order quantity.
  - Many involve personnel time.
  - The amount of work is the same no matter the size of the order.
- Carrying costs generally varies with the amount of inventory, or the order size.
  - The labor, space, and other costs increase as the order size increases.
- The actual cost of items purchased can vary if there are quantity discounts available.
The **economic order quantity** (EOQ) model is one of the oldest and most commonly known inventory control techniques.

- It is easy to use but has a number of important assumptions.
- Objective is to minimize total cost of inventory.
Economic Order Quantity

Assumptions:

1. Demand is known and constant.
2. Lead time is known and constant.
3. Receipt of inventory is instantaneous.
4. Purchase cost per unit is constant throughout the year.
5. The only variable costs are the cost of placing an order, ordering cost, and the cost of holding or storing inventory over time, holding or carrying cost, and these are constant throughout the year.
6. Orders are placed so that stockouts or shortages are avoided completely.
Inventory Usage Over Time

Order Quantity = \( Q \) = Maximum Inventory Level

Figure 6.2
Inventory Costs in the EOQ Situation

Computing Average Inventory

Average inventory level = $\frac{Q}{2}$

<table>
<thead>
<tr>
<th>DAY</th>
<th>BEGINNING</th>
<th>ENDING</th>
<th>AVERAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 1 (order received)</td>
<td>10</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>April 2</td>
<td>8</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>April 3</td>
<td>6</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>April 4</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>April 5</td>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Maximum level April 1 = 10 units
Total of daily averages = 9 + 7 + 5 + 3 + 1 = 25
Number of days = 5
Average inventory level = 25/5 = 5 units

Table 6.2
Inventory Costs in the EOQ Situation

Mathematical equations can be developed using:

\[ Q = \text{number of pieces to order} \]
\[ \text{EOQ} = Q^* = \text{optimal number of pieces to order} \]
\[ D = \text{annual demand in units for the inventory item} \]
\[ C_o = \text{ordering cost of each order} \]
\[ C_h = \text{holding or carrying cost per unit per year} \]

Annual ordering cost = \[ \left( \frac{D}{Q} \right) \times \left( \frac{C_o}{Q} \right) \]
Inventory Costs in the EOQ Situation

Mathematical equations can be developed using:

\[ Q = \text{number of pieces to order} \]
\[ \text{EOQ} = Q^* = \text{optimal number of pieces to order} \]
\[ D = \text{annual demand in units for the inventory item} \]
\[ C_o = \text{ordering cost of each order} \]
\[ C_h = \text{holding or carrying cost per unit per year} \]

Annual holding cost = \[ \left( \frac{Q}{2} \right) C_h \]

\[ = \frac{Q}{2} C_h \]
Inventory Costs in the EOQ Situation

Figure 6.3 Total Cost as a Function of Order Quantity

- Minimum Total Cost
- Curve of Total Cost of Carrying and Ordering
- Carrying Cost Curve
- Ordering Cost Curve
- Optimal Order Quantity
Finding the EOQ

According to the graph, when the EOQ assumptions are met, total cost is minimized when annual ordering cost equals annual holding cost.

\[ \frac{D}{Q} C_o = \frac{Q}{2} C_h \]

Solving for \( Q \)

\[ 2DC_o = Q^2 C_h \]

\[ \frac{2DC_o}{C_h} = Q^2 \]

\[ \sqrt{\frac{2DC_o}{C_h}} = Q = EOQ = Q^* \]
Economic Order Quantity (EOQ) Model

Summary of equations:

Annual ordering cost \( = \frac{D}{Q} C_o \)

Annual holding cost \( = \frac{Q}{2} C_h \)

\( EOQ = Q^* = \sqrt{\frac{2DC_o}{C_h}} \)
Sumco Pump Company

- Sumco Pump Company sells pump housings to other companies.
- The firm would like to reduce inventory costs by finding optimal order quantity.
  - Annual demand = 1,000 units
  - Ordering cost = $10 per order
  - Average carrying cost per unit per year = $0.50

\[
Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2(1,000)(10)}{0.50}} = \sqrt{40,000} = 200 \text{ units}
\]
Total annual cost = Order cost + Holding cost

\[ TC = \frac{D}{Q} C_o + \frac{Q}{2} C_h \]

\[ = \frac{1000}{200} (10) + \frac{200}{2} (0.5) \]

\[ = 50 + 50 = 100 \]
Total inventory cost can be written to include the cost of purchased items.

Given the EOQ assumptions, the annual purchase cost is constant at \( D \times C \) no matter the order policy, where

- \( C \) is the purchase cost per unit.
- \( D \) is the annual demand in units.

At times it may be useful to know the average dollar level of inventory:

\[
\text{Average dollar level} = \frac{(CQ)}{2}
\]
**Purchase Cost of Inventory Items**

- Inventory carrying cost is often expressed as an annual percentage of the unit cost or price of the inventory.
- This requires a new variable.

\[
I = \text{Annual inventory holding charge as a percentage of unit price or cost}
\]

- The cost of storing inventory for one year is then

\[
C_h = IC
\]

thus,

\[
Q^* = \sqrt{\frac{2DC_o}{IC}}
\]
Sensitivity Analysis with the EOQ Model

- The EOQ model assumes all values are known and fixed over time.
- Generally, however, some values are estimated or may change.
- Determining the effects of these changes is called sensitivity analysis.
- Because of the square root in the formula, changes in the inputs result in relatively small changes in the order quantity.

\[
\text{EOQ} = \sqrt{\frac{2DC_o}{C_h}}
\]
Sensitivity Analysis with the *EOQ* Model

- In the Sumco Pump example:
  \[
  \text{EOQ} = \sqrt{\frac{2(1,000)(10)}{0.50}} = 200 \text{ units}
  \]

- If the ordering cost were increased four times from $10 to $40, the order quantity would only double
  \[
  \text{EOQ} = \sqrt{\frac{2(1,000)(40)}{0.50}} = 400 \text{ units}
  \]

- In general, the *EOQ* changes by the square root of the change to any of the inputs.
Reorder Point: Determining When To Order

- Once the order quantity is determined, the next decision is **when to order**.
- The time between placing an order and its receipt is called the **lead time** ($L$) or **delivery time**.
- When to order is generally expressed as a **reorder point** ($ROP$).

\[
ROP = \left( \frac{\text{Demand per day}}{} \right) \times \left( \text{Lead time for a new order in days} \right) = d \times L
\]
Procomp’s Computer Chips

- Demand for the computer chip is 8,000 per year.
- Daily demand is 40 units.
- Delivery takes three working days.

\[ \text{ROP} = d \times L = 40 \text{ units per day} \times 3 \text{ days} = 120 \text{ units} \]

- An order based on the EOQ calculation is placed when the inventory reaches 120 units.
- The order arrives 3 days later just as the inventory is depleted.
Reorder Point Graphs

Figure 6.4

![Reorder Point Graphs](image)
EOQ Without The Instantaneous Receipt Assumption

- When inventory accumulates over time, the *instantaneous receipt* assumption does not apply.
- Daily demand rate must be taken into account.
- The revised model is often called the *production run model*.

Figure 6.5
Annual Carrying Cost for Production Run Model

- In production runs, **setup cost** replaces ordering cost.
- The model uses the following variables:

  \[ Q = \text{number of pieces per order, or production run} \]
  \[ C_s = \text{setup cost} \]
  \[ C_h = \text{holding or carrying cost per unit per year} \]
  \[ p = \text{daily production rate} \]
  \[ d = \text{daily demand rate} \]
  \[ t = \text{length of production run in days} \]
Annual Carrying Cost for Production Run Model

Maximum inventory level
\( = (\text{Total produced during the production run}) \)
\( - (\text{Total used during the production run}) \)
\( = (\text{Daily production rate})(\text{Number of days production}) \)
\( - (\text{Daily demand})(\text{Number of days production}) \)
\( = (pt) - (dt) \)

since
Total produced \( = Q = pt \)

we know
\( t = \frac{Q}{p} \)

Maximum inventory level
\( = pt - dt = p \frac{Q}{p} - d \frac{Q}{p} = Q \left( 1 - \frac{d}{p} \right) \)
Annual Carrying Cost for Production Run Model

Since the average inventory is one-half the maximum:

\[
\text{Average inventory} = \frac{Q}{2} \left( 1 - \frac{d}{p} \right)
\]

and

\[
\text{Annual holding cost} = \frac{Q}{2} \left( 1 - \frac{d}{p} \right) C_h
\]
Setup cost replaces ordering cost when a product is produced over time.

Annual setup cost \( = \frac{D}{Q} C_s \)

Annual ordering cost \( = \frac{D}{Q} C_o \)
Determining the Optimal Production Quantity

By setting setup costs equal to holding costs, we can solve for the optimal order quantity.

Annual holding cost = Annual setup cost

\[ \frac{Q}{2} \left( 1 - \frac{d}{p} \right) C_h = \frac{D}{Q} C_s \]

Solving for \( Q \), we get

\[ Q^* = \sqrt{\frac{2DC_s}{C_h \left( 1 - \frac{d}{p} \right)}} \]
Production Run Model

Summary of equations

Annual holding cost: \[ \text{Annual holding cost} = \frac{Q}{2} \left( 1 - \frac{d}{p} \right) C_h \]

Annual setup cost: \[ \text{Annual setup cost} = \frac{D}{Q} C_s \]

Optimal production quantity: \[ Q^* = \sqrt{\frac{2DC_s}{C_h \left( 1 - \frac{d}{p} \right)}} \]
Brown Manufacturing produces commercial refrigeration units in batches.

Annual demand = $D = 10,000$ units
Setup cost = $C_s = $100
Carrying cost = $C_h = $0.50 per unit per year
Daily production rate = $p = 80$ units daily
Daily demand rate = $d = 60$ units daily

1. How many units should Brown produce in each batch?
2. How long should the production part of the cycle last?
Brown Manufacturing Example

1. \[ Q^* = \sqrt{\frac{2DC_s}{C_h \left(1 - \frac{d}{p}\right)}} \]

\[ Q^* = \sqrt{\frac{2 \times 10,000 \times 100}{0.5 \left(1 - \frac{60}{80}\right)}} \]

\[ = \sqrt{\frac{2,000,000}{0.5 \left(\frac{1}{4}\right)}} = \sqrt{16,000,000} \]

\[ = 4,000 \text{ units} \]

2. Production cycle = \[ \frac{Q}{p} \]

\[ = \frac{4,000}{80} = 50 \text{ days} \]
Quantity Discount Models

- Quantity discounts are commonly available.
- The basic EOQ model is adjusted by adding in the purchase or materials cost.

Total cost = Material cost + Ordering cost + Holding cost

\[
\text{Total cost} = DC + \frac{D}{Q}C_o + \frac{Q}{2}C_h
\]

where

- \(D\) = annual demand in units
- \(C_o\) = ordering cost of each order
- \(C\) = cost per unit
- \(C_h\) = holding or carrying cost per unit per year
Quantity Discount Models

Because unit cost is now variable,

Holding cost = \( C_h = IC \)

\( I = \) holding cost as a percentage of the unit cost \( (C) \)

Total cost = \( DC + \frac{D}{Q} C_o + \frac{Q}{2} C_h \)

where

\( D = \) annual demand in units
\( C_o = \) ordering cost of each order
\( C = \) cost per unit
\( C_h = \) holding or carrying cost per unit per year
Quantity Discount Models

- A typical quantity discount schedule can look like the table below.
- However, buying at the lowest unit cost is not always the best choice.

<table>
<thead>
<tr>
<th>DISCOUNT NUMBER</th>
<th>DISCOUNT QUANTITY</th>
<th>DISCOUNT (%)</th>
<th>DISCOUNT COST ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 to 999</td>
<td>0</td>
<td>5.00</td>
</tr>
<tr>
<td>2</td>
<td>1,000 to 1,999</td>
<td>4</td>
<td>4.80</td>
</tr>
<tr>
<td>3</td>
<td>2,000 and over</td>
<td>5</td>
<td>4.75</td>
</tr>
</tbody>
</table>

Table 6.3
Quantity Discount Models

Total cost curve for the quantity discount model

Figure 6.6

Total Cost $

Order Quantity

0 1,000 2,000

TC Curve for Discount 1

TC Curve for Discount 2

TC Curve for Discount 3

EOQ for Discount 2
Brass Department Store

- Brass Department Store stocks toy race cars.
- Their supplier has given them the quantity discount schedule shown in Table 6.3.
  - Annual demand is 5,000 cars, ordering cost is $49, and holding cost is 20% of the cost of the car
- The first step is to compute EOQ values for each discount.

\[
EOQ_1 = \sqrt{\frac{(2)(5,000)(49)}{(0.2)(5.00)}} = 700 \text{ cars per order}
\]

\[
EOQ_2 = \sqrt{\frac{(2)(5,000)(49)}{(0.2)(4.80)}} = 714 \text{ cars per order}
\]

\[
EOQ_3 = \sqrt{\frac{(2)(5,000)(49)}{(0.2)(4.75)}} = 718 \text{ cars per order}
\]
Brass Department Store Example

- The second step is adjust quantities below the allowable discount range.
- The EOQ for discount 1 is allowable.
- The EOQs for discounts 2 and 3 are outside the allowable range and have to be adjusted to the smallest quantity possible to purchase and receive the discount:

\[ Q_1 = 700 \]
\[ Q_2 = 1,000 \]
\[ Q_3 = 2,000 \]
The third step is to compute the total cost for each quantity.

<table>
<thead>
<tr>
<th>DISCOUNT NUMBER</th>
<th>UNIT PRICE ((C))</th>
<th>ORDER QUANTITY ((Q))</th>
<th>ANNUAL MATERIAL COST ($) (= DC)</th>
<th>ANNUAL ORDERING COST ($) (= (D/Q)C_o)</th>
<th>ANNUAL CARRYING COST ($) (= (Q/2)C_h)</th>
<th>TOTAL ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5.00</td>
<td>700</td>
<td>25,000</td>
<td>350.00</td>
<td>350.00</td>
<td>25,700.00</td>
</tr>
<tr>
<td>2</td>
<td>4.80</td>
<td>1,000</td>
<td>24,000</td>
<td>245.00</td>
<td>480.00</td>
<td>24,725.00</td>
</tr>
<tr>
<td>3</td>
<td>4.75</td>
<td>2,000</td>
<td>23,750</td>
<td>122.50</td>
<td>950.00</td>
<td>24,822.50</td>
</tr>
</tbody>
</table>

The final step is to choose the alternative with the lowest total cost.

Table 6.4
Use of Safety Stock

- If demand or the lead time are uncertain, the exact ROP will not be known with certainty.
- To prevent stockouts, it is necessary to carry extra inventory called safety stock.
- Safety stock can prevent stockouts when demand is unusually high.
- Safety stock can be implemented by adjusting the ROP.
Use of Safety Stock

- The basic ROP equation is

\[ \text{ROP} = d \times L \]

- A safety stock variable is added to the equation to accommodate uncertain demand during lead time

\[ \text{ROP} = d \times L + SS \]

where

\[ SS = \text{safety stock} \]
Use of Safety Stock

Figure 6.7
ROP with Known Stockout Costs

- With a fixed EOQ and an ROP for placing orders, stockouts can only occur during lead time.
- Our objective is to find the safety stock quantity that will minimize the total of stockout cost and holding cost.
- We need to know the stockout cost per unit and the probability distribution of demand during lead time.
- Estimating stockout costs can be difficult as there may be direct and indirect costs.
Safety Stock with Unknown Stockout Costs

- There are many situations when stockout costs are unknown.
- An alternative approach to determining safety stock levels is to use a service level.
- A service level is the percent of time you will not be out of stock of a particular item.

Service level = 1 – Probability of a stockout
or
Probability of a stockout = 1 – Service level
**Safety Stock with the Normal Distribution**

ROP = (Average Demand During Lead Time) + Z$\sigma_{dLT}$

Safety Stock = $Z\sigma_{dLT}$

**Where:**

$Z$ = number of standard deviations for a given service level
$\sigma_{dLT}$ = standard deviation of demand during lead time
Inventory demand during lead time is normally distributed.

Mean demand during lead time is 350 units with a standard deviation of 10.

The company wants stockouts to occur only 5% of the time.

\[
\begin{align*}
\mu &= \text{Mean demand} = 350 \\
\sigma_{dLT} &= \text{Standard deviation} = 10 \\
X &= \text{Mean demand} + \text{Safety stock} \\
SS &= \text{Safety stock} = X - \mu = Z\sigma \\
Z &= \frac{X - \mu}{\sigma}
\end{align*}
\]

Figure 6.8
Hinsdale Company Example

- From Appendix A we find $Z = 1.65 = \frac{X - \mu}{\sigma} = \frac{SS}{\sigma}$

- Solving for safety stock:

$$SS = 1.65(10) = 16.5 \text{ units, or 17 units}$$

$$X = \text{ROP} = 350 + 16.5 = 366.5 \text{ units}$$

Figure 6.9
Hinsdale Company

- Different safety stock levels will be generated for different service levels.
- However, the relationship is not linear.
  - You should be aware of what a service level is costing in terms of carrying the safety stock in inventory.
- The relationship between $Z$ and safety stock can be developed as follows:

1. We know that $Z = \frac{X - \mu}{\sigma}$
2. We also know that $SS = X - \mu$
3. Thus $Z = \frac{SS}{\sigma}$
4. So we have $SS = Z \sigma = Z(10)$
### Hinsdale Company

#### Safety Stock at different service levels

<table>
<thead>
<tr>
<th>SERVICE LEVEL (%)</th>
<th>Z VALUE FROM NORMAL CURVE TABLE</th>
<th>SAFETY STOCK (UNITS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>1.28</td>
<td>12.8</td>
</tr>
<tr>
<td>91</td>
<td>1.34</td>
<td>13.4</td>
</tr>
<tr>
<td>92</td>
<td>1.41</td>
<td>14.1</td>
</tr>
<tr>
<td>93</td>
<td>1.48</td>
<td>14.8</td>
</tr>
<tr>
<td>94</td>
<td>1.55</td>
<td>15.5</td>
</tr>
<tr>
<td>95</td>
<td>1.65</td>
<td>16.5</td>
</tr>
<tr>
<td>96</td>
<td>1.75</td>
<td>17.5</td>
</tr>
<tr>
<td>97</td>
<td>1.88</td>
<td>18.8</td>
</tr>
<tr>
<td>98</td>
<td>2.05</td>
<td>20.5</td>
</tr>
<tr>
<td>99</td>
<td>2.33</td>
<td>23.3</td>
</tr>
<tr>
<td>99.99</td>
<td>3.72</td>
<td>37.2</td>
</tr>
</tbody>
</table>

Table 6.5
Service level versus annual carrying costs

This graph was developed for a specific case, but the general shape of the curve is the same for all service-level problems.
Calculating Lead Time Demand and Standard Deviation

There are three situations to consider:

- Demand is variable but lead time is constant.
- Demand is constant but lead time is variable.
- Both demand and lead time are variable.
Calculating Lead Time Demand and Standard Deviation

Demand is variable but lead time is constant:

\[ ROP = \bar{d}L + Z(\sigma_d\sqrt{L}) \]

Where:

\[ \bar{d} = \text{average daily demand} \]
\[ \sigma_d = \text{standard deviation of daily demand} \]
\[ L = \text{lead time in days} \]
Calculating Lead Time Demand and Standard Deviation

Demand is constant but lead time is variable:

\[ ROP = d\bar{L} + Z(d\sigma_L) \]

Where:

\[ \bar{L} = \text{average lead time} \]

\[ \sigma_L = \text{standard deviation of lead time} \]

\[ d = \text{daily demand} \]
Calculating Lead Time Demand and Standard Deviation

Both demand and lead time are variable.

\[ ROP = \bar{d}L + Z\left(\sqrt{L\sigma_d^2 + \bar{d}^2\sigma_L^2}\right) \]

Notice that this is the most general case and that the other two cases can be derived from this formula.
Suppose for product SKU F5402, daily demand is normally distributed, with a mean of 15 units and a standard deviation of 3. Lead time is exactly 4 days. To maintain a 97% service level, what is the ROP, and how much safety stock should be carried?

\[
ROP = \bar{d}L + Z(\sigma_d\sqrt{L})
\]

\[
ROP = 15(4) + 1.88(3\times2)
\]

\[
= 60 + 11.28
\]

\[
= 71.28
\]

So the average demand during lead time is 60 units, and safety stock is 11.28 units.
Suppose for product SKU B7319, daily demand is constant at 25 units per day, but lead time is normally distributed, with a mean of 6 days and a standard deviation of 3. To maintain a 98% service level, what is the ROP?

\[
ROP = d\bar{L} + Z(d\sigma_L)
\]

\[
ROP = 25(6) + 2.05(25\times3)
\]

\[
= 150 + 153.75
\]

\[
= 303.75
\]

So the average demand during lead time is 150 units, and safety stock is 153.75 units.
Hinsdale Company

Suppose for product SKU F9004, daily demand is normally distributed, with a mean of 20 units and a standard deviation of 4. Lead time is normally distributed, with a mean of 5 days and a standard deviation of 2 days. To maintain a 94% service level, what is the ROP?

\[ ROP = \bar{d}L + Z(\sqrt{L\sigma_d^2 + \bar{d}^2\sigma_L^2}) \]

\[ ROP = 20(5) + 1.55(\sqrt{5(16) + 400(4)}) \]

\[ ROP = 100 + 1.55(40.99) \]

\[ = 163.53 \]
Calculating Annual Holding Cost with Safety Stock

- Under standard assumptions of EOQ, average inventory is just $Q/2$.
- So annual holding cost is: $(Q/2) \cdot C_h$.
- This is not the case with safety stock because safety stock is not meant to be drawn down.
Calculating Annual Holding Cost with Safety Stock

Total annual holding cost = holding cost of regular inventory + holding cost of safety stock

\[ THC = \frac{Q}{2} C_h + (SS) C_h \]

Where:

- THC = total annual holding cost
- \( Q \) = order quantity
- \( C_h \) = holding cost per unit per year
- SS = safety stock
The purpose of ABC analysis is to divide the inventory into three groups based on the overall inventory value of the items.

Group A items account for the major portion of inventory costs.
- Typically about 70% of the dollar value but only 10% of the quantity of items.
- Forecasting and inventory management must be done carefully.

Group B items are more moderately priced.
- May represent 20% of the cost and 20% of the quantity.

Group C items are very low cost but high volume.
- It is not cost effective to spend a lot of time managing these items.
Summary of ABC Analysis

<table>
<thead>
<tr>
<th>INVENTORY GROUP</th>
<th>DOLLAR USAGE (%)</th>
<th>INVENTORY ITEMS (%)</th>
<th>ARE QUANTITATIVE CONTROL TECHNIQUES USED?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>70</td>
<td>10</td>
<td>Yes</td>
</tr>
<tr>
<td>B</td>
<td>20</td>
<td>20</td>
<td>In some cases</td>
</tr>
<tr>
<td>C</td>
<td>10</td>
<td>70</td>
<td>No</td>
</tr>
</tbody>
</table>

Table 6.8
Dependent Demand: The Case for Material Requirements Planning

- All the inventory models discussed so far have assumed demand for one item is independent of the demand for any other item.
- However, in many situations items demand is dependent on demand for one or more other items.
- In these situations, Material Requirements Planning (MRP) can be employed effectively.
Some of the benefits of MRP are:

1. Increased customer service levels.
2. Reduced inventory costs.
4. Higher total sales.
5. Faster response to market changes and shifts.
6. Reduced inventory levels without reduced customer service.

Most MRP systems are computerized, but the basic analysis is straightforward.
To achieve greater efficiency in the production process, organizations have tried to have less in-process inventory on hand. This is known as **JIT inventory**. The inventory arrives just in time to be used during the manufacturing process. One technique of implementing JIT is a manual procedure called **kanban**.
Kanban in Japanese means “card.”

With a dual-card kanban system, there is a conveyance kanban, or C-kanban, and a production kanban, or P-kanban.

Kanban systems are quite simple, but they require considerable discipline.

As there is little inventory to cover variability, the schedule must be followed exactly.
4 Steps of Kanban

1. A user takes a container of parts or inventory along with its C-kanban to his or her work area. When there are no more parts or the container is empty, the user returns the container along with the C-kanban to the producer area.

2. At the producer area, there is a full container of parts along with a P-kanban. The user detaches the P-kanban from the full container and takes the container and the C-kanban back to his or her area for immediate use.
4 Steps of Kanban

3. The detached P-kanban goes back to the producer area along with the empty container. The P-kanban is a signal that new parts are to be manufactured or that new parts are to be placed in the container and is attached to the container when it is filled.

4. This process repeats itself during the typical workday.
The Kanban System

Figure 6.16
MRP has evolved to include not only the materials required in production, but also the labor hours, material cost, and other resources related to production.

In this approach the term MRP II is often used and the word resource replaces the word requirements.

As this concept evolved and sophisticated software was developed, these systems became known as enterprise resource planning (ERP) systems.
The objective of an ERP System is to reduce costs by integrating all of the operations of a firm. Starts with the supplier of materials needed and flows through the organization to include invoicing the customer of the final product.

Data are entered only once into a database where it can be quickly and easily accessed by anyone in the organization.

Benefits include:
- Reduced transaction costs.
- Increased speed and accuracy of information.
Enterprise Resource Planning

- **Drawbacks to ERP:**
  - The software is expensive to buy and costly to customize.
    - Small systems can cost hundreds of thousands of dollars.
    - Large systems can cost hundreds of millions.
  - The implementation of an ERP system may require a company to change its normal operations.
  - Employees are often resistant to change.
  - Training employees on the use of the new software can be expensive.